

The Effective Lagrangian for Bulk Fermions

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Abstract

We compute the dimension 6 effective Lagrangian arising from the tree level integration of an arbitrary number of bulk fermions in models with warped extra dimensions. The coefficients of the effective operators are written in terms of simple integrals of the metric and are valid for arbitrary warp factors, with or without an infrared brane, and for a general Higgs profile. All relevant tree level fermion effects in electroweak and flavor observables can be computed using this effective Lagrangian.

I. INTRODUCTION

Models with warped extra dimensions [1] offer a calculable path to the study of electroweak symmetry breaking (EWSB) induced by a strongly coupled sector [2]. Even if EWSB does not proceed precisely through the dual of the original Randall-Sundrum model, it is plausible that models with arbitrary warp factors provide us with a general enough sampling of the parameter space of strong EWSB models to give us the insight needed to decipher the LHC data. It is therefore crucial to develop techniques that allow us to study the phenomenological implications of models with warped extra dimensions for arbitrary backgrounds.

Indirect constraints on models with warped extra dimensions have been extensively studied in the past (see for instance [3–5]). Most of these studies are performed using a fixed background, and the few cases in which the low energy effective Lagrangian is computed for an arbitrary background, only the contribution of bulk gauge bosons has been taken into account. Fermion contributions on the other hand have been computed only for a fixed background, typically for a finite number of KK modes and always for specific models (using general results from integration of four-dimensional vector-like fermions [6, 7], see also [8]). The purpose of this work is to fill this gap by computing the dimension 6 effective Lagrangian generated after the integration at tree level of an arbitrary number of bulk fermions, including the effect of the full tower. The result is written in terms of integrals of the metric and the Higgs profile, thus being applicable to models with arbitrary warp factor and Higgs localization. EWSB effects can be included to all orders essentially only when they enter directly or effectively as boundary terms. In order to consider a more general case, we assume a light Standard Model (SM)-like Higgs with an arbitrary profile. This is a very good approximation in general if there is a light Higgs in the spectrum. For instance in composite Higgs models, bounds on the S parameter typically imply suppressed non-linear Higgs terms [4]. In order to obtain the background independent results, we have used 5D propagators to integrate out the full tower of bulk fermions (these techniques can be also useful for 5D loop calculations, see for instance [9]). Holographic methods would give the same results (see [10] for a detailed comparison of both methods).

The rest of the article is organized as follows. We introduce our notation and discuss how to integrate out bulk fermions using 5D propagators in Section II. The different terms

entering the coefficients of the effective Lagrangian are computed in terms of integrals of the metric and Higgs profiles in Section III. An application of the formalism developed in these two sections is discussed in Section IV, in which we compute the corrections to SM Yukawa couplings induced by bulk fermions. We conclude in Section V. Technical details on the calculation and results of the 5D propagators and the general expression for the SM gauge and Yukawa couplings derived from the effective Lagrangian are given in two appendices.

II. INTEGRATING OUT BULK FERMIONS

In this section we will compute the effective Lagrangian resulting from the tree level integration of an arbitrary number of bulk fermions in models with warped extra dimensions. The only relevant assumption we make is to have a light Higgs in the spectrum, responsible for electroweak symmetry breaking, but otherwise remain completely general. Our methods can be generalized to include the effects of EWSB to all orders by including the Higgs vev as part of our general background or through boundary conditions (this is essential for instance in Higgsless models [11]). For clarity, we prefer to consider EWSB perturbatively as non-linear effects in the Higgs vev do not interfere with the effects of bulk fermions that we are interested in at the level of dimension 6 operators. Also, we will focus in this work on the quark sector but the results presented here are applicable, with minimal changes to the leptonic sector too (see [7]). With our assumption on EWSB, all fields can be classified according to their SM quantum numbers. In this case, the only new quarks that contribute to the effective Lagrangian at tree level and dimension 6 are the ones with the quantum numbers shown in table I.

$Q^{(m)}$	U	D	$\begin{pmatrix} U \\ D \end{pmatrix}$	$\begin{pmatrix} X \\ U \end{pmatrix}$	$\begin{pmatrix} D \\ Y \end{pmatrix}$	$\begin{pmatrix} X \\ U \\ D \end{pmatrix}$	$\begin{pmatrix} U \\ D \\ Y \end{pmatrix}$
Notation			Q	Q^X	Q^Y	T^X	T^Y
$SU(2)_L \times U(1)_Y$	$0_{\frac{2}{3}}$	$0_{-\frac{1}{3}}$	$2_{\frac{1}{6}}$	$2_{\frac{7}{6}}$	$2_{-\frac{5}{6}}$	$3_{\frac{2}{3}}$	$3_{-\frac{1}{3}}$

TABLE I: New quark multiplets, $Q^{(m)}$, that can mix with the SM fermions through Yukawa couplings. The index m labels the differen types of fermion multiplet additions in the given order. The electric charge is the sum of the third component of isospin T_3 and the hypercharge Y .

We consider a 5D space with a general warped metric

$$ds^2 = a^2(z) [\eta_{\mu\nu} dx^\mu dx^\nu - dz^2], \quad (1)$$

where x^μ denote the standard four extended space-time dimensions and z parameterizes the extra dimension, which has two boundaries, $L_0 \leq z \leq L_1$, called the UV and IR branes, respectively. The warp factor is general but normalized to $a(L_0) = 1$. Popular choices for this warp factor are $a(z) = 1$ for flat space and $a(z) = L_0/z$ for AdS₅. Also, the case of no IR brane can be simply obtained by taking $L_1 \rightarrow \infty$.

Let us consider a number of bulk fermions, $\Psi^b(x, z)$, classified according to their SM quantum numbers. Bulk fermions with quantum numbers other than the ones in table I do not contribute to the effective Lagrangian we are computing and are disregarded. We separate them into their zero mode (when it exists) and non-zero mode components

$$\Psi_{L,R}^b(x, z) = f_{iL,R}^{(0)b}(z) \psi_{L,R}^{(0)i}(x) + \tilde{\Psi}_{L,R}^b(x, z), \quad (2)$$

where, from now on, repeated indices imply summation unless otherwise stated, $i = 1, 2, 3$ denotes the SM fermion generation index, and we have explicitly written the (4D) chiralities

$$\Psi_{L,R} = \mathcal{P}_{L,R} \Psi, \quad \mathcal{P}_{L,R} \equiv \frac{1 \mp \gamma^5}{2}. \quad (3)$$

The action can then be written

$$S_5 = S_{SM} + \Delta S, \quad (4)$$

where the part of the action involving only zero modes gives, after integration over the extra dimension, the SM action S_{SM} , for which we take for following convention,

$$S_{SM} = \int d^4x \left\{ \bar{q}_L^i i \not{D} q_L^i + \bar{u}_R^i i \not{D} u_R^i + \bar{d}_R^i i \not{D} d_R^i - (V_{ij}^\dagger \lambda_j^u \bar{q}_L^i \tilde{\phi} u_R^j + \lambda_i^d \bar{q}_L^i \phi d_R^i + \text{h.c.}) \right\}, \quad (5)$$

whereas the part of the action involving heavy modes can be written

$$\Delta S = \int d^4x \int_{L_0}^{L_1} dz \left\{ \bar{\tilde{\Psi}}^b \left(\mathcal{O}_K^b \delta^{bc} + \mathcal{O}_\varphi^{bc} \Phi^{bc} \right) \tilde{\Psi}^c + \left[\bar{\tilde{\Psi}}^b J^b + \text{h.c.} \right] \right\}, \quad (6)$$

where \mathcal{O}_K represents the kinetic (including the SM gauge bosons through the covariant derivative) and mass contributions and \mathcal{O}_φ the couplings to the SM Higgs. Φ^{bc} represents here the appropriate expression of the SM Higgs, according to the quantum numbers of the fermions involved. Explicitly we have

$$\begin{aligned} \overline{[2_Y]}[1_{Y+\frac{1}{2}}] &\rightarrow \Phi \equiv \tilde{\phi}, & \overline{[3_Y]}[2_{Y+\frac{1}{2}}] &\rightarrow \Phi \equiv \phi^\dagger \frac{\sigma^a}{2}, \\ \overline{[2_Y]}[1_{Y-\frac{1}{2}}] &\rightarrow \Phi \equiv \phi, & \overline{[3_Y]}[2_{Y-\frac{1}{2}}] &\rightarrow \Phi \equiv \tilde{\phi}^\dagger \frac{\sigma^a}{2}, \end{aligned} \quad (7)$$

where we have denoted in squared brackets the quantum numbers of the two fermionic fields involved in the coupling, $\tilde{\phi} \equiv i\sigma_2\phi^*$ denotes the $Y = -1/2$ Higgs doublet and the Pauli matrices are in the basis of $T_3^L = \pm 1, 0$, as are the corresponding quark triplets. Finally, the currents J , which contain a SM fermion and the SM Higgs, have the generic form

$$J^b = J_L^b + J_R^b = \mathcal{O}_\varphi^{bc} \Phi^{bc} [f_{sL}^{(0)c} \psi_L^{(0)s} + f_{sR}^{(0)c} \psi_R^{(0)s}]. \quad (8)$$

Due to chirality of the SM spectrum, only one of the two components is non-vanishing for each type of heavy fermion. In particular, using the notation in Table I, the non-vanishing currents can be written as

$$J_L^U = -\lambda_{Uq_L^j}(z) V_{ji} \tilde{\phi}^\dagger q_L^i, \quad J_L^D = -\lambda_{Dq_L^i}(z) \phi^\dagger q_L^i, \quad (9)$$

$$J_R^Q = -\lambda_{Qu_R^i}(z) \tilde{\phi} u_R^i - \lambda_{Qd_R^i}(z) \phi d_R^i, \quad (10)$$

$$J_R^{Q^x} = -\lambda_{Q^x u_R^i}(z) \phi u_R^i, \quad J_R^{Q^y} = -\lambda_{Q^y d_R^i}(z) \tilde{\phi} d_R^i, \quad (11)$$

$$J_L^{T^x} = -\lambda_{T^x q_L^j}(z) V_{ji} \frac{\sigma^a}{2} \tilde{\phi}^\dagger q_L^i, \quad J_L^{T^y} = -\lambda_{T^y q_L^j}(z) V_{ji} \frac{\sigma^a}{2} \phi^\dagger q_L^i, \quad (12)$$

where the $\lambda_{\Psi\psi}(z)$ encode the extra dimensional dependence of the effective Yukawa couplings. All these operators are functions of z that encode the dependence on the metric and the wave functions of the different fields. Their explicit expressions will be given in the next section. Their mass dimensions are $[\mathcal{O}_K] = 1$, $[\mathcal{O}_\varphi] = 0$, $[J] = 3$ (also $[\Psi] = 2$).

We can integrate out the heavy fields at tree level by solving their classical equations of motion

$$[\mathcal{O}_K^b \delta^{bc} + \mathcal{O}_\varphi^{bc} \Phi^{bc}] \tilde{\Psi}^c = -J^b, \quad (13)$$

and inserting the solution back in the action

$$\Delta S = \int d^4x dz \bar{J}^b(x, z) \tilde{\Psi}^b(x, z) = \int \frac{d^4p}{(2\pi)^4} dz \bar{J}^a(p, z) \tilde{\Psi}^b(p, z), \quad (14)$$

where in the last equality we have switched to mixed position/momentum space [12] by Fourier transforming with respect to the four extended dimensions. We have implicitly denoted the functions and their Fourier transforms by their argument. $\tilde{\Psi}^b$ is supposed to be replaced with the solution of the equation of motion which, in mixed position/momentum space reads

$$\mathcal{O}_K^b(p, z) \tilde{\Psi}^b(p, z) = -J^b(p, z) - \int \frac{d^4p_1}{(2\pi)^4} \mathcal{O}_\varphi^{bc}(z) \Phi^{bc}(p_1) \tilde{\Psi}^c(p - p_1, z). \quad (15)$$

Strictly speaking, one has to include the terms in \mathcal{O}_K containing the SM gauge bosons as a convolution on the right hand side. Gauge invariance guarantees however that we can forget about those terms and recover them at the end by turning normal derivatives to covariant ones. We have explicitly checked that both methods give the same result. Let us define the Green's function of \mathcal{O}_K with the zero mode subtracted

$$\mathcal{O}_K^b(p, z)P_p^{bc}(z, z') = \delta^{bc}\delta(z - z'), \quad \tilde{P}^{bc} = P^{bc} - P_{\text{zero mode}}^{bc}, \quad (16)$$

which can be decomposed in their chiral (from the 4D point of view) components [13]

$$P_{LL} = \mathcal{P}_L P \mathcal{P}_R, \quad P_{LR} = \mathcal{P}_L P \mathcal{P}_L, \quad P_{RR} = \mathcal{P}_R P \mathcal{P}_L, \quad P_{RL} = \mathcal{P}_R P \mathcal{P}_R. \quad (17)$$

The different components can be written as follows

$$\begin{aligned} \tilde{P}_{LL}^{ab}(p; z, z') &= (\not{p}L_1)\hat{P}_{LL}^{ab}(p; z, z'), & \tilde{P}_{RR}^{ab}(p; z, z') &= (\not{p}L_1)\hat{P}_{RR}^{ab}(p; z, z'), \\ \tilde{P}_{LR}^{ab}(p; z, z') &= \hat{P}_{LR}^{ab}(p; z, z'), & \tilde{P}_{RL}^{ab}(p; z, z') &= \hat{P}_{RL}^{ab}(p; z, z'), \end{aligned} \quad (18)$$

with $\hat{P}(p; z, z') = \hat{P}^{(0)}(z, z') + (p^2 L_1^2) \hat{P}^{(1)}(z, z') + \dots$, where we have introduced the appropriate powers of L_1 to keep all components of the Green's functions with the same (vanishing) mass dimension. The solution of the equation of motion can be written in terms of these Green's functions by iteration

$$\begin{aligned} \tilde{\Psi}^b(p, z) &= - \int dz' \tilde{P}_p^{bc}(z, z') \left\{ J^c(p, z') + \int \frac{d^4 p_1}{(2\pi)^4} \mathcal{O}_\phi^{cd}(z') \Phi^{cd}(p_1) \tilde{\Psi}^d(p - p_1, z') \right\} \\ &= - \int dz' \tilde{P}_p^{bc}(z, z') \left\{ J^c(p, z') \right. \\ &\quad \left. - \int dz'' \int \frac{d^4 p_1}{(2\pi)^4} \mathcal{O}_\phi^{cd}(z') \Phi^{cd}(p_1) \tilde{P}_{p-p_1}^{de}(z', z'') J^e(p - p_1, z'') \right\} + \dots \end{aligned} \quad (19)$$

Inserting this solution back in the action and taking into account the following properties

$$P^{bc} \neq 0 \Rightarrow \bar{J}_L^b J_R^c = \bar{J}_R^b J_L^c = 0, \quad (20)$$

$$\mathcal{O}_\phi^{bc} \neq 0 \Rightarrow \bar{J}_L^b J_L^c = \bar{J}_R^b J_R^c = 0, \quad (21)$$

we get the following effective Lagrangian

$$\begin{aligned} \mathcal{L}_6 &= -L_1 \int dz dz' \left\{ \bar{J}_L^b(x, z) \hat{P}_{RR}^{(0)bc}(z, z') i \not{D} J_L^c(x, z') + (L \leftrightarrow R) \right\} \\ &\quad + \int dz dz' dz'' \left\{ \bar{J}_L^b(x, z) \hat{P}_{RL}^{(0)bc}(z, z') \mathcal{O}_\phi^{cd}(z') \Phi^{cd}(x) \hat{P}_{RL}^{(0)de}(z', z'') J_R^e(x, z'') + \text{h.c.} \right\}, \end{aligned} \quad (22)$$

where we have switched back to position space (including the terms coming from the SM gauge fields), factored out the integration over 4D space to obtain the effective Lagrangian and the terms that we have not written are either absent due to the chirality of the SM spectrum or give rise to operators of dimension higher than 6.

In order to write our effective Lagrangian in the standard basis of [14] we need to manipulate the operators in Eq. (22). After applying the Leibniz rule for the covariant derivative, using the equations of motion from \mathcal{L}_{SM} for the SM fermions and Fierz reordering, we end up with the following effective Lagrangian [6]

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{SM} + \frac{1}{\Lambda^2} \left[\sum_{\psi_L} (\alpha_{\phi\psi_L}^{(3)})_{ij} (\mathcal{O}_{\phi\psi_L}^{(3)})^{ij} + \sum_{\psi} (\alpha_{\phi\psi}^{(1)})_{ij} (\mathcal{O}_{\phi\psi}^{(1)})^{ij} \right. \\ \left. + (\alpha_{\phi\phi})_{ij} (\mathcal{O}_{\phi\phi})^{ij} + \sum_{\psi_R} (\alpha_{\psi_R\phi})_{ij} (\mathcal{O}_{\psi_R\phi})^{ij} + \text{h.c.} \right], \quad (23)$$

where we have explicitly factored out two powers of the cut-off $\Lambda \sim L_1^{-1}$ (so that the coefficients of the different operators are dimensionless), the sums run over all left-handed (LH) SM fields ψ_L , all SM fields ψ and all SM right-handed (RH) fields ψ_R , respectively. The different operators are defined as follows

$$(\mathcal{O}_{\phi\psi_L}^{(3)})^{ij} = (\phi^\dagger \sigma^I i D_\mu \phi) (\bar{\psi}_L^i \sigma^I \gamma^\mu \psi_L^j), \quad (24)$$

$$(\mathcal{O}_{\phi\psi}^{(1)})^{ij} = (\phi^\dagger i D_\mu \phi) (\bar{\psi}^i \gamma^\mu \psi^j), \quad (25)$$

$$(\mathcal{O}_{\phi\phi})^{ij} = (\phi^\dagger \epsilon i D_\mu \phi) (\bar{u}_R^i \gamma^\mu d_R^j), \quad (26)$$

$$(\mathcal{O}_{\psi_R\phi})^{ij} = (\phi^\dagger \phi) (\bar{\psi}_L^i \Phi_{\psi_R} \psi_R^j), \quad (27)$$

where $\Phi_{u_R} = \tilde{\phi}$ and $\Phi_{d_R} = \phi$. The first three kinds of operators come from the first line in Eq. (22) and involve contributions from one kind of multiplet in table I at a time. The last operator, on the other hand, receives corrections from both lines in Eq. (22) and therefore involves one or two different heavy multiplets at a time. The resulting coefficients of the effective operators are collected in tables II and III, written in terms of the following overlap integrals

$$\beta_{\psi^i \psi'^j}^{\Psi; \chi} \equiv -L_1 \int dz dz' \lambda_{\psi^i \Psi^b}^\dagger(z) \hat{P}_{\chi\chi}^{(0)\Psi^b \Psi^c}(z, z') \lambda_{\Psi^c \psi'^j}(z'), \quad (28)$$

$$\gamma_{\psi^i \psi'^j}^{\Psi \Psi'; \chi \chi'} \equiv \int dz dz' dz'' \lambda_{\psi^i \Psi^b}^\dagger(z) \hat{P}_{\chi\chi'}^{(0)\Psi^b \Psi^c}(z, z') \mathcal{O}_\varphi^{\Psi^c \Psi'^d}(z') \hat{P}_{\chi\chi'}^{(0)\Psi'^d \Psi'^e}(z', z'') \lambda_{\Psi'^e \psi'^j}(z''). \quad (29)$$

The only remaining pieces to compute the effective Lagrangian are the explicit expressions

of the $\mathcal{O}_\varphi(z)$ and $\lambda(z)$ functions and the zero momentum propagators. The former two are done in the next section whereas the latter are given in Appendix A.

TABLE II: Coefficients α_x^m resulting from the integration of an arbitrary number of each type of bulk quark. The coefficients $\beta_{\psi^i\psi'^j}^{\Psi;\chi}$ are defined in Eq.(28).

$Q^{(m)}$	$\frac{(\alpha_{\phi q}^{(1)})_{ij}}{\Lambda^2}$	$\frac{(\alpha_{\phi q}^{(3)})_{ij}}{\Lambda^2}$	$\frac{(\alpha_{\phi u})_{ij}}{\Lambda^2}$	$\frac{(\alpha_{\phi d})_{ij}}{\Lambda^2}$	$\frac{(\alpha_{\phi\phi})_{ij}}{\Lambda^2}$	$\frac{(\alpha_{u\phi})_{ij}}{\Lambda^2}$	$\frac{(\alpha_{d\phi})_{ij}}{\Lambda^2}$
U	$\frac{1}{4}V_{ik}^\dagger\beta_{q_L^k q_L^l}^{U;R}V_{lj}$	$-\frac{(\alpha_{\phi q}^{(1)})_{ij}}{\Lambda^2}$	—	—	—	$2\frac{(\alpha_{\phi q}^{(1)})_{ik}}{\Lambda^2}V_{kj}^\dagger\lambda_j^u$	—
D	$-\frac{1}{4}\beta_{q_L^i q_L^j}^{D;R}$	$\frac{(\alpha_{\phi q}^{(1)})_{ij}}{\Lambda^2}$	—	—	—	—	$-2\frac{(\alpha_{\phi q}^{(1)})_{ij}}{\Lambda^2}\lambda_j^d$
Q	—	—	$-\frac{1}{2}\beta_{u_R^i u_R^j}^{Q;L}$	$\frac{1}{2}\beta_{d_R^i d_R^j}^{Q;L}$	$-\beta_{u_R^i d_R^j}^{Q;L}$	$-V_{ik}^\dagger\lambda_k^u\frac{(\alpha_{\phi u})_{kj}}{\Lambda^2}$	$\lambda_i^d\frac{(\alpha_{\phi d})_{ij}}{\Lambda^2}$
Q^X	—	—	$\frac{1}{2}\beta_{u_R^i u_R^j}^{Q^X;L}$	—	—	$V_{ik}^\dagger\lambda_k^u\frac{(\alpha_{\phi u})_{kj}}{\Lambda^2}$	—
Q^Y	—	—	—	$-\frac{1}{2}\beta_{d_R^i d_R^j}^{Q^Y;L}$	—	—	$-\lambda_i^d\frac{(\alpha_{\phi d})_{ij}}{\Lambda^2}$
T^X	$\frac{3}{16}V_{ik}^\dagger\beta_{q_L^k q_L^l}^{T^X;R}V_{lj}$	$\frac{1}{3}\frac{(\alpha_{\phi q}^{(1)})_{ij}}{\Lambda^2}$	—	—	—	$\frac{2}{3}\frac{(\alpha_{\phi q}^{(1)})_{ik}}{\Lambda^2}V_{kj}^\dagger\lambda_j^u$	$\frac{4}{3}\frac{(\alpha_{\phi q}^{(1)})_{ij}}{\Lambda^2}\lambda_j^d$
T^Y	$-\frac{3}{16}V_{ik}^\dagger\beta_{q_L^k q_L^l}^{T^Y;R}V_{lj}$	$-\frac{1}{3}\frac{(\alpha_{\phi q}^{(1)})_{ij}}{\Lambda^2}$	—	—	—	$-\frac{4}{3}\frac{(\alpha_{\phi q}^{(1)})_{ik}}{\Lambda^2}V_{kj}^\dagger\lambda_j^u$	$-\frac{2}{3}\frac{(\alpha_{\phi q}^{(1)})_{ij}}{\Lambda^2}\lambda_j^d$

TABLE III: Coefficients α_x^{mn} resulting from mixing between bulk multiplets. The coefficients $\gamma_{\psi^i\psi'^j}^{\Psi\Psi';\chi\chi'}$ are defined in Eq. (29).

	U, Q	U, Q^X	D, Q	D, Q^Y	Q, T^X	Q, T^Y	Q^X, T^X	Q^Y, T^Y
$\frac{(\alpha_{u\phi}^{mn})_{ij}}{\Lambda^2}$	$V_{ik}^\dagger\gamma_{q_L^k u_R^j}^{UQ;RL}$	$V_{ik}^\dagger\gamma_{q_L^k u_R^j}^{UQ^X;RL}$	—	—	$\frac{1}{4}V_{ik}^\dagger\gamma_{q_L^k u_R^j}^{T^X Q;RL}$	$\frac{1}{2}V_{ik}^\dagger\gamma_{q_L^k u_R^j}^{T^Y Q;RL}$	$-\frac{1}{4}V_{ik}^\dagger\gamma_{q_L^k u_R^j}^{T^X Q^X;RL}$	—
$\frac{(\alpha_{d\phi}^{mn})_{ij}}{\Lambda^2}$	—	—	$\gamma_{q_L^i d_R^j}^{DQ;RL}$	$\gamma_{q_L^i d_R^j}^{DQ^Y;RL}$	$\frac{1}{2}V_{ik}^\dagger\gamma_{q_L^k d_R^j}^{T^X Q;RL}$	$\frac{1}{4}V_{ik}^\dagger\gamma_{q_L^k d_R^j}^{T^Y Q;RL}$	—	$-\frac{1}{4}V_{ik}^\dagger\gamma_{q_L^k d_R^j}^{T^Y Q^Y;RL}$

III. GENERAL EXPRESSION FOR THE EFFECTIVE COEFFICIENTS

Let us now give the explicit form of the different terms entering the coefficients of the effective Lagrangian. The relevant part of the action for the bulk fermions is given by

$$S = \int d^4x dz a^4 \bar{\Psi}^b \left\{ \left[i\not{D} + \left(\partial_z + 2\frac{a'}{a} \right) \gamma^5 - aM_{\Psi^b} \right] \delta^{bc} - a f_h \lambda_{bc}^{(5)} \Phi^{bc} \right\} \Psi^c + S_{\text{bound.}}, \quad (30)$$

where $S_{\text{bound.}}$ denotes any possible boundary term, b, c run over all bulk multiplets with quantum numbers appearing in table I, D denotes the SM covariant derivative, we have assumed a bulk Higgs with a zero mode profile given by $f_h(z)$, normalized to

$$\int_{L_0}^{L_1} dz a^3 f_h^2 = 1, \quad (31)$$

$\lambda_{bc}^{(5)}$ denote the 5D Yukawa couplings and Φ^{bc} represents the correct combination of the SM Higgs appropriate for the quantum numbers of $\Psi_{b,c}$ as described in Eq. (7). In the expressions above, we have assumed the Higgs to be a 5D scalar. In models of Gauge-Higgs unification, the Higgs comes from a gauge boson and the replacement $f_h \rightarrow a^{-1} f_h$ should be made. Subtleties related to a boundary Higgs will be discussed in section IV. This action can be simplified with the following field redefinition

$$\Psi^b \rightarrow a^{-2} a_{M_b}^{-\gamma^5/2} \Psi^b, \quad (32)$$

where we have defined the effective (mass dependent) metric

$$a_{M_b}(z) \equiv \exp \left[-2 \int_{L_0}^z dz' a(z') M_{\Psi_b}(z') \right] = a_{-M_b}^{-1}(z). \quad (33)$$

After this field redefinition, the action can be written

$$S = \int d^4x dz \bar{\Psi}^b \left[\left(i \not{D} a_{M_b}^{-\gamma^5/2} + \gamma^5 \partial_z \right) \delta^{bc} - a a_{M_b}^{\gamma^5/2} f_h \lambda_{bc}^{(5)} \Phi_{bc} a_{M_c}^{-\gamma^5/2} \right] \Psi^c + S_{\text{bound.}}. \quad (34)$$

We separate now the bulk fermions into their zero mode and non-zero mode components as in Eq.(2)

$$\Psi_{L,R}^b(x, z) = f_{iL,R}^{(0)b} \psi_{L,R}^{(0)i}(x) + \tilde{\Psi}_{L,R}^b(x, z). \quad (35)$$

These zero modes realize the SM fermionic spectrum. We have denoted with i the flavor of the canonically normalized fermion zero modes. Note that we have allowed these fermion zero modes to be shared by several different bulk fields, as happens when boundary conditions mix different bulk fields. In the basis after the redefinition (32) the zero mode profiles are just constants fixed by the boundary conditions and the orthonormality condition

$$f_{q_L^i}^{Q\dagger} \left[\int dz a_{M_Q} \right] f_{q_L^j}^Q = f_{u_R^i}^{U\dagger} \left[\int dz a_{M_U}^{-1} \right] f_{u_R^j}^U = f_{d_R^i}^{D\dagger} \left[\int dz a_{M_D}^{-1} \right] f_{d_R^j}^D = \delta_{ij}, \quad (36)$$

where as usual, a sum over all possible values of Q , U or D is understood. We can further require that the zero mode profiles diagonalize the zero mode Yukawa couplings for the down

sector and the right part of the Yukawa couplings for the up sector. This amounts to

$$\begin{aligned} f_{q_L^i}^{Q\dagger} \left[\int dz a f_h a_{\frac{M_Q}{2}} \lambda_{QU}^{(5)} a_{-\frac{M_U}{2}} \right] f_{u_R^j}^U &= V_{ij}^\dagger \lambda_j^u, \\ f_{q_L^i}^{Q\dagger} \left[\int dz a f_h a_{\frac{M_Q}{2}} \lambda_{QD}^{(5)} a_{-\frac{M_D}{2}} \right] f_{d_R^j}^D &= \delta_{ij} \lambda_j^d. \end{aligned} \quad (37)$$

Note that for every zero mode profile that satisfy Eq. (36) we can define new ones

$$f_{q_L^i}^Q \rightarrow f_{q_L^i}^Q V_{ij}^L, \quad f_{u_R^i}^U \rightarrow f_{u_R^i}^U V_{ij}^{uR}, \quad f_{d_R^i}^D \rightarrow f_{d_R^i}^D V_{ij}^{dR}, \quad (38)$$

with V^L, V^{uR}, V^{dR} 3×3 unitary matrices so that the new profiles satisfy both (36) and (37). This separation allows us to write the action in the form of Eq. (4), where Eqs. (36-37) guarantee that the SM action satisfies the convention in Eq. (5). The quadratic operator reads

$$\mathcal{O}_K^b = \left[i \not{D} a_{M_b}^{-\gamma^5} + \gamma^5 \partial_z \right] + \text{bound. terms}, \quad (39)$$

from which we can compute the relevant propagators as discussed in Appendix A. The Higgs operator is given by

$$\mathcal{O}_\varphi^{bc} = -a a_{M_b}^{\gamma^5/2} f_h \lambda_{bc}^{(5)} a_{M_c}^{-\gamma^5/2} + \text{bound. terms}. \quad (40)$$

Finally, the explicit expressions for the non-vanishing currents read

$$\lambda_{Uq_L^i}(z) = \sum_Q a f_h a_{M_u}^{-\frac{1}{2}} \lambda_{UQ}^{(5)} a_{M_Q}^{\frac{1}{2}} f_{q_L^j}^Q V_{ji}^\dagger, \quad (41)$$

$$\lambda_{Dq_L^i}(z) = \sum_Q a f_h a_{M_D}^{-\frac{1}{2}} \lambda_{DQ}^{(5)} a_{M_Q}^{\frac{1}{2}} f_{q_L^j}^Q, \quad (42)$$

$$\lambda_{Qu_R^i}(z) = \sum_U a f_h a_{M_Q}^{\frac{1}{2}} \lambda_{QU}^{(5)} a_{M_U}^{-\frac{1}{2}} f_{u_R^j}^U, \quad (43)$$

$$\lambda_{Qd_R^i}(z) = \sum_D a f_h a_{M_Q}^{\frac{1}{2}} \lambda_{QD}^{(5)} a_{M_D}^{-\frac{1}{2}} f_{d_R^j}^D, \quad (44)$$

$$\lambda_{Q^x u_R^i}(z) = \sum_U a f_h a_{M_{Q^x}}^{\frac{1}{2}} \lambda_{Q^x U}^{(5)} a_{M_U}^{-\frac{1}{2}} f_{u_R^j}^U, \quad (45)$$

$$\lambda_{Q^y d_R^i}(z) = \sum_D a f_h a_{M_{Q^y}}^{\frac{1}{2}} \lambda_{Q^y D}^{(5)} a_{M_D}^{-\frac{1}{2}} f_{d_R^j}^D, \quad (46)$$

$$\lambda_{T^x q_L^i}(z) = \sum_Q a f_h a_{M_{T^x}}^{-\frac{1}{2}} \lambda_{T^x Q}^{(5)} a_{M_Q}^{\frac{1}{2}} f_{q_L^j}^Q V_{ji}^\dagger, \quad (47)$$

$$\lambda_{T^y q_L^i}(z) = \sum_Q a f_h a_{M_{T^y}}^{-\frac{1}{2}} \lambda_{T^y Q}^{(5)} a_{M_Q}^{\frac{1}{2}} f_{q_L^j}^Q V_{ji}^\dagger, \quad (48)$$

where possible contributions to boundary Yukawa couplings have not been explicitly written.

We can now plug these explicit expressions, Eqs.(40-48) and (A14-A29) in Eqs. (28,29) to compute the coefficients of the general effective Lagrangian for an arbitrary warp factor and Higgs profile. As a example of the use of these general equations we will compute in the next section the effect of bulk fermions in SM quark Yukawa couplings.

IV. APPLICATION: HIGGS COUPLINGS

As a straight-forward application of our formalism we compute flavour violating Yukawa couplings in the down sector of a simplified model with two bulk fermions with the quantum numbers of Q and D (following the notation in Table I) and boundary conditions $Q \sim [++]$, $D \sim [--]$. The two signs denote the boundary condition at the UV and IR brane, respectively, and a $+$ ($-$) denotes Dirichlet boundary conditions for the RH (LH) component of the bulk field at the corresponding brane. Their zero modes give rise to the SM q_L and d_R quarks. We want to compute the modified SM Yukawa couplings that result after the integration of the heavy fields. This has been studied recently in Refs. [15, 16]. In particular, Ref. [15] clarifies the implications of brane-localized Yukawa couplings for fields with Dirichlet boundary conditions. As we will see, the use of 5D propagators makes it apparent the ambiguity of these couplings and a very simple regularization of the brane terms gives directly the same result as previously obtained with other methods.¹ The SM fermion gauge and Yukawa couplings derived from the effective Lagrangian in Eq. (23) were computed in [6]. We collect the main results in Appendix B. The modified Yukawa couplings can be written, in the physical basis (*i.e.* with diagonal fermion masses including effects of order v^2/Λ^2)

$$\mathcal{L}^H = -\frac{1}{\sqrt{2}}(\bar{u}_L^i Y_{ij}^u u_R^j + \bar{d}_L^i Y_{ij}^d d_R^j)H + \text{h.c.}, \quad (49)$$

¹ Similar ambiguities occur in brane localized kinetic terms for fields with Dirichlet boundary conditions [17]. In that case, there is a well-defined prescription to deal with such ambiguities in terms of field redefinitions and classical renormalization [18].

where the coefficients read

$$\begin{aligned} Y_{ij}^u &= \delta_{ij} \lambda_j^u - \frac{v^2}{\Lambda^2} \left(V_{ik} (\alpha_{u\phi})_{kj} + \frac{1}{4} \delta_{ij} [V_{ik} (\alpha_{u\phi})_{kj} + (\alpha_{u\phi})_{ik}^\dagger V_{kj}^\dagger] \right), \\ Y_{ij}^d &= \delta_{ij} \lambda_j^d - \frac{v^2}{\Lambda^2} \left((\alpha_{d\phi})_{ij} + \frac{1}{4} \delta_{ij} (\alpha_{d\phi} + \alpha_{d\phi}^\dagger)_{ij} \right), \end{aligned} \quad (50)$$

with $v \approx 246$ GeV the Higgs vacuum expectation value. Note that we have not written couplings proportional to $\partial_\mu H$ because they vanish in our case due to the hermiticity of the corresponding operator coefficients (see Appendix B). These modified couplings are thus fully determined by two coefficients that read, in our example,

$$\frac{V_{ik} (\alpha_{u\phi})_{kj}}{\Lambda^2} = \frac{1}{2} \lambda_i^u \beta_{u_R^i u_R^j}^{Q;L}, \quad (51)$$

$$\frac{(\alpha_{d\phi})_{ij}}{\Lambda^2} = \frac{1}{2} \beta_{q_L^i q_L^j}^{D;R} \lambda_j^u + \frac{1}{2} \lambda_i^d \beta_{d_R^i d_R^j}^{Q;L} + \gamma_{q_L^i d_R^j}^{DQ;RL}. \quad (52)$$

Using the explicit expressions for the propagators in Appendix A, we can compute the corrections to the Yukawa couplings for arbitrary backgrounds and Higgs profiles. In the case of an IR boundary Higgs, the terms proportional to β contain only Yukawa couplings of fields with Neumann boundary conditions. They are well defined in the thin brane limit and a direct application of our formulae reproduces the results in the literature. The term proportional to γ on the other hand, involves a Yukawa coupling of fields with Dirichlet boundary conditions and therefore should *a priori* vanish. We will show that the result is ambiguous in the thin brane limit but a simple regularization of the brane gives a result that agrees with the ones presented in the literature.

In order to directly compare with the results in the literature we consider a pure AdS₅ background, a boundary localized Higgs and a constant bulk mass for the fermions $M = c/L_0$. Following [15] we define the Yukawa Lagrangian for a boundary Higgs (for simplicity we focus on the down sector)

$$S_{\text{Yuk}} = - \int d^4 x dz \delta(z - L_1) a^3 \left[\bar{Q}_L Y_1^{5D} L_0 \phi D_R + \bar{Q}_R Y_2^{5D} L_0 \phi D_L + \text{h.c.} \right] + \dots, \quad (53)$$

where we have already canonically normalized the Higgs boson ϕ (so that its vev is $v \sim L_1^{-1}$) and we have included a term involving boundary values of fields with Dirichlet boundary conditions. The latter terms do not involve the zero modes and therefore do not affect the values of the effective z -dependent Yukawa couplings appearing in the currents, that read

in our case

$$\lambda_{Dq}(z) = \delta(z - L_1) a_{M_D}^{-\frac{1}{2}} (Y_1^{5D})^\dagger L_1 a_{M_Q}^{\frac{1}{2}} f_{q_L}^Q, \quad (54)$$

$$\lambda_{Qd}(z) = \delta(z - L_1) a_{M_Q}^{\frac{1}{2}} Y_1^{5D} L_1 a_{M_D}^{-\frac{1}{2}} f_{d_R}^D. \quad (55)$$

However, both terms contribute in principle to the \mathcal{O}_φ^{bc} term, which reads in our case

$$\mathcal{O}_\varphi^{DQ}(z) = -\delta(z - L_1) L_1 \left[a_{M_D}^{-\frac{1}{2}} Y_1^{5D} a_{M_Q}^{\frac{1}{2}} \mathcal{P}_L + a_{M_D}^{\frac{1}{2}} Y_2^{5D} a_{M_Q}^{-\frac{1}{2}} \mathcal{P}_R \right]. \quad (56)$$

In fact, due to the chiral structure of the coefficient $\gamma_{q_L^i d_R^j}^{DQ;RL}$ only the term proportional to Y_2 can give a non-vanishing contribution. Now that we have the explicit expression for the relevant overlaps with the Higgs, we only need the corresponding propagators to compute the modified Yukawa couplings. The β coefficients are given by

$$\beta_{q_L^i q_L^j}^{D;R} = -L_1^3 f_{q_L}^Q a_{M_Q}^{\frac{1}{2}} Y_1^{5D} a_{M_D}^{-\frac{1}{2}} \hat{P}_{RR}^{(0)[-]}(c_D, L_1, L_1) a_{M_D}^{-\frac{1}{2}} (Y_1^{5D})^\dagger a_{M_Q}^{\frac{1}{2}} f_{q_L}^Q, \quad (57)$$

$$\beta_{d_R^i d_R^j}^{Q;L} = -L_1^3 f_{d_R}^D a_{M_D}^{-\frac{1}{2}} Y_1^{5D} a_{M_Q}^{\frac{1}{2}} \hat{P}_{LL}^{(0)[+]}(c_Q, L_1, L_1) a_{M_Q}^{\frac{1}{2}} Y_1^{5D} a_{M_D}^{-\frac{1}{2}} f_{d_R}^D, \quad (58)$$

where $a_M(L_1) = (L_0/L_1)^{2ML_0}$, $c_Q = M_Q L_0$, $c_D = M_D L_0$ and the explicit expressions of the boundary propagators for the AdS case is

$$\begin{aligned} \hat{P}_{LL}^{(0)[+]}(c, L_1, L_1) &= \hat{P}_{RR}^{(0)[-]}(-c, L_1, L_1) = \frac{L_0^2}{(4c^2 - 4c - 3)[1 - (L_0/L_1)^{2c-1}]^2} \times \\ &\left[(4c^2 - 4c - 3) \left(\frac{L_0}{L_1} \right)^{-2} - (2c - 1)^2 + (3 - 2c) \left(\frac{L_0}{L_1} \right)^{-2c-1} + (1 + 2c) \left(\frac{L_0}{L_1} \right)^{2c-3} \right]. \end{aligned} \quad (59)$$

See Eqs. (A14) and (A23). The γ coefficient on the other hand, involves propagators that present discontinuities at the branes

$$\hat{P}_{RL}^{(0)[+]}(z, z') = \theta(z - z') - \frac{1}{L_M} \int_{L_0}^z dz_1 a_M(z_1), \quad (60)$$

and

$$\hat{P}_{RL}^{(0)[-]}(z, z') = -\theta(z' - z) + \frac{1}{L_{-M}} \int_{L_0}^{z'} dz_1 a_{-M}(z_1). \quad (61)$$

This discontinuities indicate that the coefficients γ are ambiguous in the thin brane limit and therefore the Dirichlet brane Yukawa term, proportional to Y_2 , can potentially give a non-vanishing contribution. In order to obtain a finite result we will replace the brane terms in Eqs. (54-56) with regulated delta functions

$$\delta(z - L_1) \rightarrow \delta_\epsilon(z - L_1) = \begin{cases} 0, & z < L_1 - \epsilon, \\ \frac{1}{\epsilon}, & L_1 - \epsilon \leq z \leq L_1. \end{cases} \quad (62)$$

Once the branes are regulated, we can perform all the relevant integrals and then take the limit $\epsilon \rightarrow 0$. The (non-vanishing) result we obtain is

$$\gamma_{q_L^i d_R^j}^{DQ;RL} = \frac{L_1^3}{3} f_{q_L}^Q a_{M_Q}^{\frac{1}{2}} Y_1^{5D} Y_2^{5D \dagger} Y_1^{5D} a_{M_D}^{-\frac{1}{2}} f_{d_R}^D. \quad (63)$$

Using the fact that (see Appendix B)

$$m_i^{d(\text{phys.})} \delta_{ij} - \frac{v}{\sqrt{2}} Y_{ij}^d = \frac{v^3}{\sqrt{2}} \frac{(\alpha_{d\phi})_{ij}}{\Lambda^2}, \quad (64)$$

the result for $(\alpha_{d\phi})_{ij}$ deduced from Eqs.(52,63) exactly reproduces the results of Eq. (73) in Ref. [15], taking into account the following dictionary between our notation and theirs:

$$v^{\text{here}} = \sqrt{2} v^{[15]}, \quad (a_{M_Q}^{\frac{1}{2}} f_{q_L}^Q)^{\text{here}} = (\hat{F}_q)^{[15]}, \quad (a_{M_D}^{-\frac{1}{2}} f_{d_R}^D)^{\text{here}} = (\hat{F}_d)^{[15]}. \quad (65)$$

In Ref. [15] the correct result was obtained, either by taking the boundary limit of a bulk Higgs or by summing the effect of a number of KK modes of the order of the inverse width of the regulated brane in the case of a brane Higgs. In our case, this resummation is done automatically by means of the 5D propagators which immediately shows the ambiguity present in terms of Dirichlet Yukawa couplings and the need for a regularization of such couplings.

V. CONCLUSIONS

Models with warped extra dimensions represent calculable examples of models in which EWSB proceeds through a strongly coupled conformal sector. With the LHC currently probing the TeV scale, it is important to have a handy way of computing the low energy effects of the largest possible number of different models of strong EWSB. The effective Lagrangian of models with warped extra dimensions arising from the tree level integration of bulk gauge bosons has been known for general background metrics and Higgs profiles for some time now. In this work we have completed the most relevant part of the tree level (dimension 6) effective Lagrangian by including the effect of an arbitrary number of bulk fermions, again with general warp factor and Higgs profile. Our general calculation involves the integration of the full 5D fields (with the zero modes subtracted when present) by means of the 5D propagators. This has the advantage of trading eigenvalue searches and sums with integrals that can be easily done numerically. Also, it makes apparent the subtleties related

to brane localized terms involving fields with Dirichlet boundary conditions as the automatic resummation of all the modes gives a non-vanishing propagator near the brane for certain components of fields with Dirichlet boundary conditions.

Our results are completely general and can be applied to any model with warped (or flat) extra dimensions, with or without an IR brane [19]. This finally permits the complete calculation of the low energy effects of a large variety of holographic models of strong EWSB, including all the relevant electroweak and flavor effects.

Note Added:

Upon completion of this work, Ref. [20] appeared in the arXives. In that reference, the contribution of bulk fermions to the coupling of the down sector to the Z boson is computed using similar techniques to the ones presented here.

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Appendix A: Derivation of the fermionic Green's functions

Assuming no boundary terms, the quadratic operator in mixed momentum/position space reads

$$\mathcal{O}_K = \left[\not{p} a_M^{-\gamma^5} + \gamma^5 \partial_z \right]. \quad (\text{A1})$$

The different components of the fermionic Green's functions satisfy the following coupled equations

$$a_M^{-1} \hat{P}^{RL} - L_1 \partial_z \hat{P}^{LL} = 0, \quad a_M L_1 p^2 \hat{P}^{LL} + \partial_z \hat{P}^{RL} = \delta(z - z'), \quad (\text{A2})$$

$$a_M \hat{P}^{LR} + L_1 \partial_z \hat{P}^{RR} = 0, \quad a_M^{-1} L_1 p^2 \hat{P}^{RR} - \partial_z \hat{P}^{LR} = \delta(z - z'), \quad (\text{A3})$$

and can be written in terms of the KK mode profiles as follows

$$L_1 \hat{P}_p^{LL}(z, z') = \sum_n \frac{f_n^L(z) f_n^L(z')}{p^2 - m_n^2}, \quad (\text{A4})$$

$$L_1 \hat{P}_p^{RR}(z, z') = \sum_n \frac{f_n^R(z) f_n^R(z')}{p^2 - m_n^2}, \quad (\text{A5})$$

$$\hat{P}_p^{RL}(z, z') = \hat{P}_p^{LR}(z', z) = \sum_n m_n \frac{f_n^R(z) f_n^L(z')}{p^2 - m_n^2}. \quad (\text{A6})$$

We can obtain equations for the LL and RR components by iteration

$$L_1 \left[\partial_z a_M \partial_z + a_M p^2 \right] \hat{P}_p^{LL}(z, z') = \delta(z - z'), \quad (\text{A7})$$

$$L_1 \left[\partial_z a_{-M} \partial_z + a_{-M} p^2 \right] \hat{P}_p^{RR}(z, z') = \delta(z - z'). \quad (\text{A8})$$

These equations are identical to the ones of a gauge boson with a generalized metric $a_{\pm M}(z)$ for LH and RH components, respectively. The zero momentum propagators for gauge bosons with an arbitrary warp factor can be computed in a number of ways. The most direct method is probably the one suggested recently in [5] which amounts to solving directly the equations that the Green's function *at zero momentum* satisfy. The resulting Green's functions at zero momentum, with zero modes subtracted and for an arbitrary warp factor a , read (see [5])

$$\begin{aligned} L_1 \hat{P}^{(0)[++]}(a, z, z'), &= -\frac{1}{L} \int_{L_0}^{z<} dz_2 a^{-1}(z_2) \int_{L_0}^{z_2} dz_1 a(z_1) - \frac{1}{L} \int_{z>}^{L_1} dz_2 a^{-1}(z_2) \int_{z_2}^{L_1} dz_1 a(z_1) \\ &+ \frac{1}{L^2} \int_{L_0}^{L_1} dz_1 a^{-1}(z_1) \int_{L_0}^{z_1} dz_2 a(z_2) \int_{z_1}^{L_1} dz_3 a(z_3), \end{aligned} \quad (\text{A9})$$

$$L_1 \hat{P}^{(0)[- -]}(a, z, z') = -\frac{\int_{L_0}^{z<} dz_1 a^{-1}(z_1) \int_{z>}^{L_1} dz_2 a^{-1}(z_2)}{\int_{L_0}^{L_1} dz_3 a^{-1}(z_3)}, \quad (\text{A10})$$

$$L_1 \hat{P}^{(0)[- +]}(a, z, z') = -\int_{L_0}^{z<} dz_1 a^{-1}(z_1), \quad (\text{A11})$$

$$L_1 \hat{P}^{(0)[+ -]}(a, z, z') = -\int_{z>}^{L_1} dz_1 a^{-1}(z_1), \quad (\text{A12})$$

where the superscript denotes the boundary conditions with a $+$ ($-$) denoting Neumann (Dirichlet) boundary conditions and the first (second) sign referring to the UV (IR) boundary and we have defined

$$L \equiv \int_{L_0}^{L_1} dz a(z). \quad (\text{A13})$$

The fermionic Green's functions $\hat{P}_{LL}^{(0)}$ and $\hat{P}_{RR}^{(0)}$ can be expressed in terms of these with the replacement $a \rightarrow a_{\pm M}$ (and the understanding that the boundary conditions correspond to

the LH chirality of the 5D field). The LR and RL components can then be trivially obtained from these by using the first equations in (A2) or (A3). We can also use $\hat{P}_{LR}(z, z') = \hat{P}_{RL}(z', z)$.

Let us now give the result for the different possibilities of boundary conditions.

- $[++]$ case

The LL and RR components are given by

$$\hat{P}_{LL}^{(0)}(z, z') = \hat{P}^{(0)[++]}(a_M, z, z'), \quad (\text{A14})$$

$$\hat{P}_{RR}^{(0)}(z, z') = \hat{P}^{(0)[--]}(a_{-M}, z, z'), \quad (\text{A15})$$

whereas the mixed ones read

$$\hat{P}_{LR}^{(0)}(z, z') = \theta(z' - z) - \frac{1}{L_M} \int_{L_0}^{z'} dz_1 a_M(z_1), \quad (\text{A16})$$

$$\hat{P}_{RL}^{(0)}(z, z') = \theta(z - z') - \frac{1}{L_M} \int_{L_0}^z dz_1 a_M(z_1), \quad (\text{A17})$$

- $[-+]$ case

$$\hat{P}_{LL}^{(0)}(z, z') = \hat{P}^{(0)[-+] }(a_M, z, z'), \quad (\text{A18})$$

$$\hat{P}_{RR}^{(0)}(z, z') = \hat{P}^{(0)[+-]}(a_{-M}, z, z'), \quad (\text{A19})$$

$$\hat{P}_{LR}^{(0)}(z, z') = -\theta(z - z'), \quad (\text{A20})$$

$$\hat{P}_{RL}^{(0)}(z, z') = -\theta(z' - z), \quad (\text{A21})$$

- $[--]$ case

$$\hat{P}_{LL}^{(0)}(z, z') = \hat{P}^{(0)[--]}(a_M, z, z'), \quad (\text{A22})$$

$$\hat{P}_{RR}^{(0)}(z, z') = \hat{P}^{(0)[++]}(a_{-M}, z, z'), \quad (\text{A23})$$

$$\hat{P}_{LR}^{(0)}(z, z') = -\theta(z - z') + \frac{1}{L_{-M}} \int_{L_0}^z dz_1 a_M^{-1}(z_1), \quad (\text{A24})$$

$$\hat{P}_{RL}^{(0)}(z, z') = -\theta(z' - z) + \frac{1}{L_{-M}} \int_{L_0}^{z'} dz_1 a_M^{-1}(z_1), \quad (\text{A25})$$

- $[+-]$ case

$$\hat{P}_{LL}^{(0)}(z, z') = \hat{P}^{(0)[+-]}(a_M, z, z'), \quad (\text{A26})$$

$$\hat{P}_{RR}^{(0)}(z, z') = \hat{P}^{(0)[-+] }(a_{-M}, z, z'), \quad (\text{A27})$$

$$\hat{P}_{LR}^{(0)}(z, z') = \theta(z' - z), \quad (\text{A28})$$

$$\hat{P}_{RL}^{(0)}(z, z') = \theta(z - z'). \quad (\text{A29})$$

Appendix B: Trilinear couplings in the physical basis

In this appendix we reproduce the couplings of the SM quarks to the SM gauge bosons and Higgs with the Lagrangian in Eq. (23) that were computed in [6]. The masses of the SM quarks in the physical basis read

$$m_i^{u(\text{phys.})} = \frac{v}{\sqrt{2}} \left(\lambda_i^u - \frac{1}{4} [(V\alpha_{u\phi})_{ii} + (V\alpha_{u\phi})_{ii}^\dagger] \frac{v^2}{\Lambda^2} \right), \quad (\text{B1})$$

$$m_i^{d(\text{phys.})} = \frac{v}{\sqrt{2}} \left(\lambda_i^d - \frac{1}{4} [(\alpha_{d\phi})_{ii} + (\alpha_{d\phi})_{ii}^\dagger] \frac{v^2}{\Lambda^2} \right). \quad (\text{B2})$$

In this basis, the SM quark couplings to the SM gauge and Higgs bosons can be written as

$$\begin{aligned} \mathcal{L}^Z &= -\frac{g}{2\cos\theta_W} \left(\bar{u}_L^i X_{ij}^{uL} \gamma^\mu u_L^j + \bar{u}_R^i X_{ij}^{uR} \gamma^\mu u_R^j \right. \\ &\quad \left. - \bar{d}_L^i X_{ij}^{dL} \gamma^\mu d_L^j - \bar{d}_R^i X_{ij}^{dR} \gamma^\mu d_R^j - 2\sin^2\theta_W J_{\text{EM}}^\mu \right) Z_\mu, \\ \mathcal{L}^W &= -\frac{g}{\sqrt{2}} (\bar{u}_L^i W_{ij}^L \gamma^\mu d_L^j + \bar{u}_R^i W_{ij}^R \gamma^\mu d_R^j) W_\mu^+ + h.c., \\ \mathcal{L}^H &= -\frac{1}{\sqrt{2}} (\bar{u}_L^i Y_{ij}^u u_R^j + \bar{d}_L^i Y_{ij}^d d_R^j + h.c.) H \\ &\quad + (\bar{u}_L^i Z_{ij}^{uL} \gamma^\mu u_L^j + \bar{u}_R^i Z_{ij}^{uR} \gamma^\mu u_R^j - \bar{d}_L^i Z_{ij}^{dL} \gamma^\mu d_L^j - \bar{d}_R^i Z_{ij}^{dR} \gamma^\mu d_R^j) i\partial_\mu H. \end{aligned} \quad (\text{B3})$$

The unbroken $U(1)_Q$ protects the terms proportional to J_{EM}^μ . The expressions to order $1/\Lambda^2$ of the coupling matrices X , W , Y and Z in terms of the coefficients α_x are:

$$\begin{aligned}
X_{ij}^{uL} &= \delta_{ij} - \frac{1}{2} \frac{v^2}{\Lambda^2} V_{ik} (\alpha_{\phi q}^{(1)} + \alpha_{\phi q}^{(1)\dagger} - \alpha_{\phi q}^{(3)} - \alpha_{\phi q}^{(3)\dagger})_{kl} V_{lj}^\dagger, \\
X_{ij}^{uR} &= -\frac{1}{2} \frac{v^2}{\Lambda^2} (\alpha_{\phi u} + \alpha_{\phi u}^\dagger)_{ij}, \\
X_{ij}^{dL} &= \delta_{ij} + \frac{1}{2} \frac{v^2}{\Lambda^2} (\alpha_{\phi q}^{(1)} + \alpha_{\phi q}^{(1)\dagger} + \alpha_{\phi q}^{(3)} + \alpha_{\phi q}^{(3)\dagger})_{ij}, \\
X_{ij}^{dR} &= \frac{1}{2} \frac{v^2}{\Lambda^2} (\alpha_{\phi d} + \alpha_{\phi d}^\dagger)_{ij}, \\
W_{ij}^L &= \tilde{V}_{ik} \left(\delta_{kj} + \frac{v^2}{\Lambda^2} (\alpha_{\phi q}^{(3)})_{kj} \right), \\
W_{ij}^R &= -\frac{1}{2} \frac{v^2}{\Lambda^2} (\alpha_{\phi\phi})_{ij}, \\
Y_{ij}^u &= \delta_{ij} \lambda_j^u - \frac{v^2}{\Lambda^2} \left(V_{ik} (\alpha_{u\phi})_{kj} + \frac{1}{4} \delta_{ij} [V_{ik} (\alpha_{u\phi})_{kj} + (\alpha_{u\phi}^\dagger)_{ik} V_{kj}^\dagger] \right), \\
Y_{ij}^d &= \delta_{ij} \lambda_j^d - \frac{v^2}{\Lambda^2} \left((\alpha_{d\phi})_{ij} + \frac{1}{4} \delta_{ij} (\alpha_{d\phi} + \alpha_{d\phi}^\dagger)_{ij} \right), \\
Z_{ij}^{uL} &= -\frac{1}{2} \frac{v}{\Lambda^2} V_{ik} (\alpha_{\phi q}^{(1)} - \alpha_{\phi q}^{(1)\dagger} - \alpha_{\phi q}^{(3)} + \alpha_{\phi q}^{(3)\dagger})_{kl} V_{lj}^\dagger, \\
Z_{ij}^{uR} &= -\frac{1}{2} \frac{v}{\Lambda^2} (\alpha_{\phi u} - \alpha_{\phi u}^\dagger)_{ij}, \\
Z_{ij}^{dL} &= \frac{1}{2} \frac{v}{\Lambda^2} (\alpha_{\phi q}^{(1)} - \alpha_{\phi q}^{(1)\dagger} + \alpha_{\phi q}^{(3)} - \alpha_{\phi q}^{(3)\dagger})_{ij}, \\
Z_{ij}^{dR} &= \frac{1}{2} \frac{v}{\Lambda^2} (\alpha_{\phi d} - \alpha_{\phi d}^\dagger)_{ij}.
\end{aligned} \tag{B4}$$

We have introduced the unitary matrix

$$\tilde{V} = V + \frac{v^2}{\Lambda^2} (V A_L^d - A_L^u V), \tag{B5}$$

with

$$\begin{aligned}
(A_L^u)_{ij} &= \frac{1}{2} \left(1 - \frac{1}{2} \delta_{ij} \right) \frac{\lambda_i^u (V \alpha_{u\phi})_{ij}^\dagger + (-1)^{\delta_{ij}} (V \alpha_{u\phi})_{ij} \lambda_j^u}{(\lambda_i^u)^2 - (-1)^{\delta_{ij}} (\lambda_j^u)^2}, \\
(A_L^d)_{ij} &= \frac{1}{2} \left(1 - \frac{1}{2} \delta_{ij} \right) \frac{\lambda_i^d (\alpha_{d\phi})_{ij}^\dagger + (-1)^{\delta_{ij}} (\alpha_{d\phi})_{ij} \lambda_j^d}{(\lambda_i^d)^2 - (-1)^{\delta_{ij}} (\lambda_j^d)^2}.
\end{aligned} \tag{B6}$$

Note that, to order $1/\Lambda^2$, we can substitute V by \tilde{V} everywhere in Eq. (B4), so that the different couplings depend on only one unitary matrix.

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